Inverse function

1) one-to-one function

2) onto function

3) one-to-one correspondence function

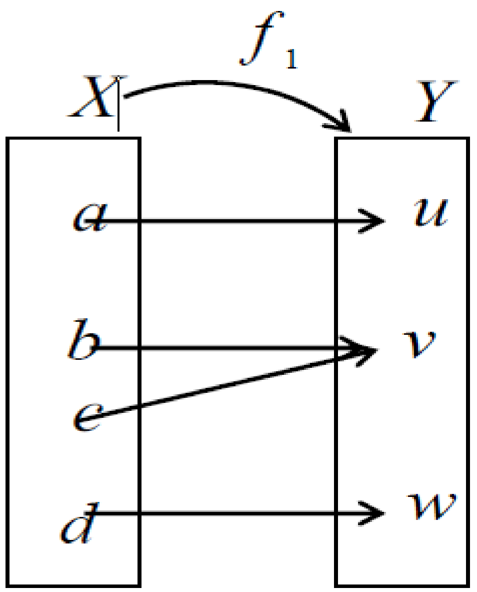
Definition:

The function $f : X \to Y$ is said to be one-to-one function if for all $x_1, x_2 \in X = D_f$

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

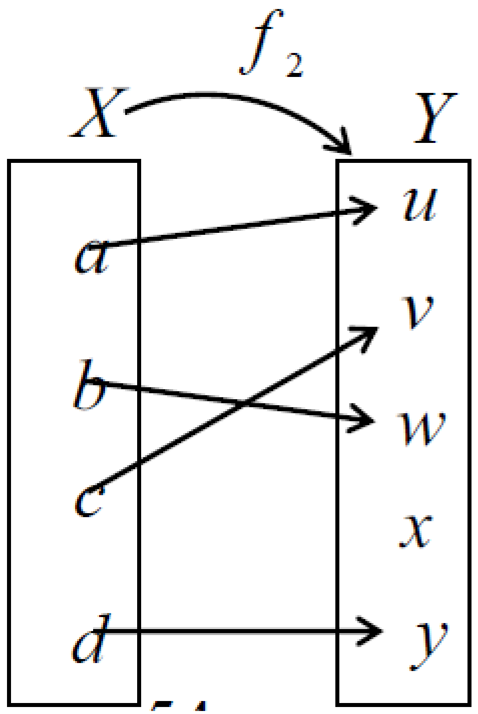
or

$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) .$$



(1) The function f_1 is not one-to-one, because

$$f_1(b) = f_1(c) = v \text{ but } b \neq c .$$

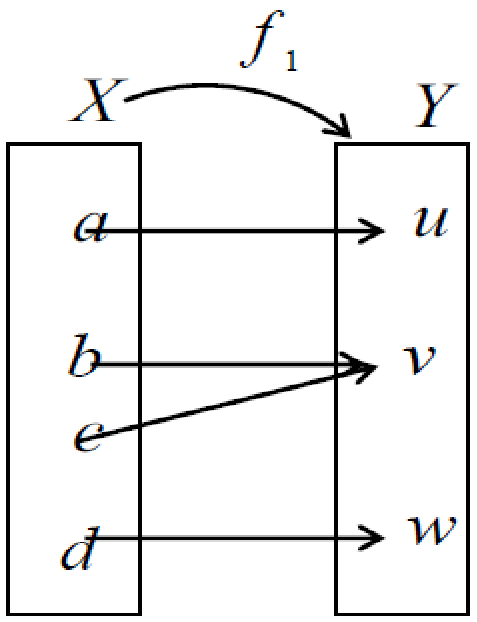


(2) The function f_2 is one-to-one, because for all $x_1, x_2 \in X = D_{f_2}$

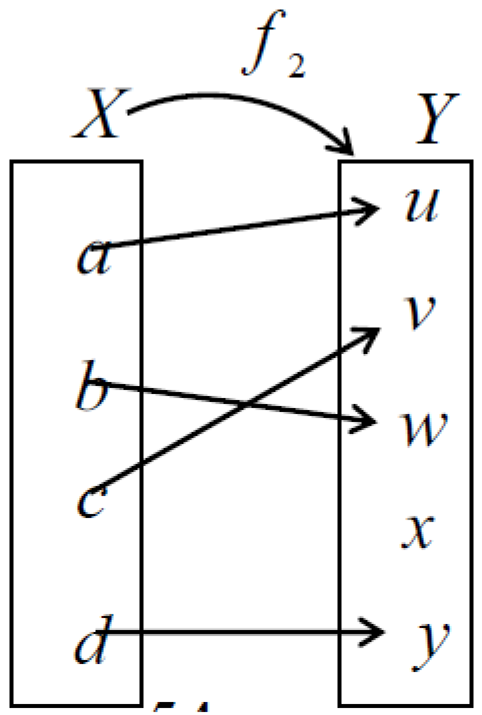
$$f_2(x_1) = f_2(x_2) \Rightarrow x_1 = x_2 .$$

Definition:

The function $f : X \to Y$, $y = f(x)$ is said to be onto function if for all $y \in Y$ there exists $x \in X$ such that $y = f(x)$. That is the function is onto if $R_f = Y$.



1 The function f_1 is onto, because $R_{f_1} = \{u, v, w\} = Y$.

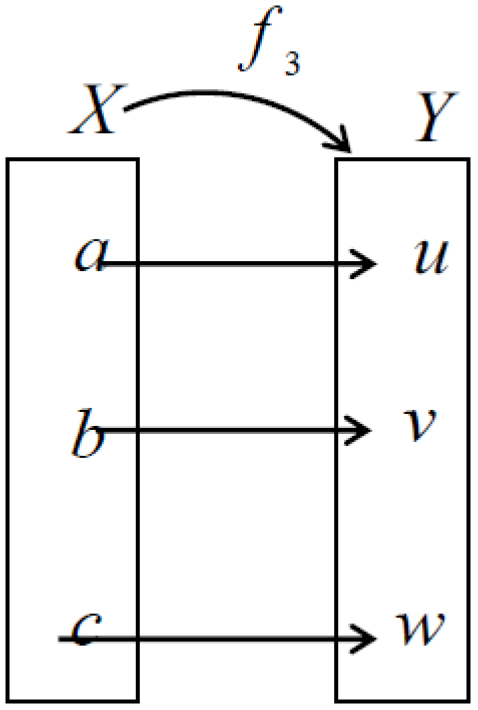


2 The function f_2 is not onto, because

$$R_{f_2} = \{u, v, w, y\} \neq Y .$$

Definition:

The function $f : X \rightarrow Y$, $y = f(x)$ is said to be one-to-one correspondence function if the function is one-to-one and onto.



(3) The function f_3 is one-to-one, because for all $x_1, x_2 \in X = D_{f_3}$

$$f_3(x_1) = f_3(x_2) \Rightarrow x_1 = x_2 .$$

The function f_3 is onto, because $R_{f_3} = \{u, v, w\} = Y$.

Then f_3 is one-to-one correspondence.

Example:

Determine whether each function is one-to-one, onto or one-to-one correspondence

$$1)\ f : \mathrm{R} \to \mathrm{R}\ , \quad f(x) = 2x + 3$$

Solution:

(1) $f : \mathbb{R} \to \mathbb{R}\ ,\ \ f(x) = 2x + 3$

$\forall\ x_1, x_2 \in D_f = \mathbb{R}$

$f(x_1) = f(x_2) \Rightarrow 2x_1 + 3 = 2x_2 + 3 \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$

then the function f is one-to-one.

Since $R_f = \mathbb{R} =$ co-domain, then the function f is onto.

(2) $f:\mathbb{R}\to\mathbb{R},\ f(x)=x^2$

$\forall x_1, x_2 \in D_f = \mathbb{R}$

$f(x_1)=f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

then the function f is not one-to-one, for example

$f(-1)=f(1)=1$ but $-1\neq 1$.

Since $R_f = [0,\infty[\ \neq \mathbb{R}$, then the function f is not onto.

Definition:

If the function $f: X \to Y$, $y = f(x)$ one-to-one correspondence, the function has an inverse function, denote f^{-1}, such that $f^{-1}: Y \to X$ and $x = f^{-1}(y)$.

1) $f: R \to R$, $f(x) = 2x + 3$ 1-1 correspondence

$$y = 2x + 3$$
$$2x = y - 3$$
$$x = \frac{1}{2}(y - 3)$$

$$f^{-1}: R \to R, \qquad f^{-1}(y) = \frac{1}{2}(y - 3)$$